Nordea

DGBi23

Exposure to real rates and nominal rates

The optimal hedge of the linker depends on the correlation between nominal and real rates and the drivers of nominal rates going forward….

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Markets

Dual duration – linker and nominal bond asymmetry

- The relation between nominal rates and real rates and inflation expectations is given by the Fisher equation
- For a nominal bond it does not matter if the movement in rates comes from the real rate component or the inflation component
- For the inflation linked bonds (ILB) it does make a difference, however. The ILB has a real duration which can be observed in the market, but the ILB's inflation exposure is zero by construction
- If inflation rises then the coupons and principal are scaled by the realized inflation. This means that $\frac{\partial PV}{\partial \pi} = 0$
- As can be seen from the example, an increase in nominal rates that stem from increased real rates hits both bonds. Whereas an increase in nominal rates that is based on increased inflation expectations **only** hits the nominal bond

$$Fisher: i = r + \pi$$

Nominal bond: $dPV = \frac{\partial PV}{\partial i} \cdot di$

ILB: $dPV = \frac{\partial PV}{\partial r} \cdot dr + \frac{\partial PV}{\partial \pi} \cdot d\pi$

An example

DK 1.5 15Nov23 duration: 9.14

DK IL 0.1 15Nov23 duration: 9.87

1) **Nominal rates** increase 100bps, $\boldsymbol{\pi}$ unchanged (r up)

Nom: $dPV = \frac{\partial PV}{\partial i} \cdot \text{di} = -0.0914 * 100 = \mathbf{-9.14}$

ILB: $dPV = \frac{\partial PV}{\partial r} \cdot dr + \frac{\partial PV}{\partial \pi} \cdot d\pi \quad \rightarrow$

$dPV = -0.0987 * 100 + 0 * 0 = \mathbf{-9.87}$

2) **Nominal rates** increase 100bps, **r** unchanged (π up)

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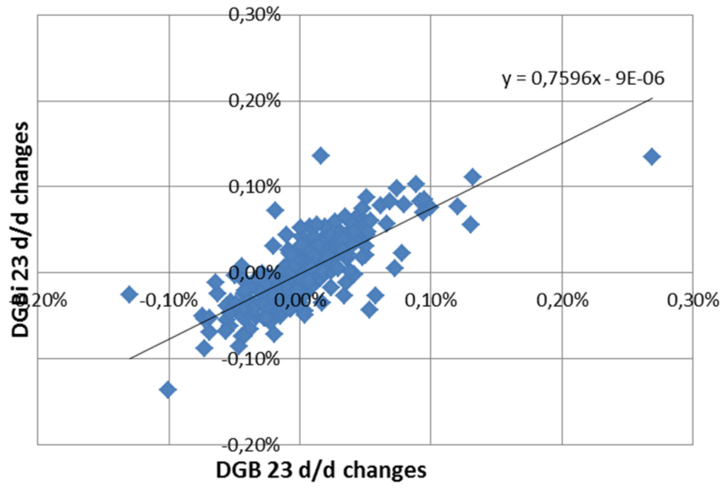
Dual duration– what is the exposure in nominal terms?

- Using the chain rule we can establish the link between the PV of the ILB and nominal rates. $\frac{\partial PV}{\partial r}$ can be observed in the market and $\frac{\partial r}{\partial i}$ can be estimated from historical data
- If we calculate the linear relation of d/d changes in a 1y window of nominal and real rates we get 0.76. Using the chain rule we get $ModD_i$= $\frac{\partial PV}{\partial r} \cdot \frac{\partial r}{\partial i}$= -9.87 · 0.76 = -7.5
- This is the nominal exposure assuming that the relation between real- and nominal rates is stable. The equivalent nominal duration is used to compare the nominal duration of nominal bonds and ILB. A similar calculation on 10y German bonds also gives a correlation 0.76
- In other words, if you want to hedge the nominal exposure on an ILB you should buy 100 DGBi and sell 100 · 7.5/ 9.87 = 75 nominal bonds

Differentiation using the chain rule

$$\text{Chain rule: } \frac{\partial PV}{\partial i} = \frac{\partial PV}{\partial r} \cdot \frac{\partial r}{\partial i}$$

Extracting the real rates and nominal rates correlation



Pure break-even inflation exposure

- Exposure to higher BEI is taken by buying the linker and selling the cash bond
- What is the hedge ratio between nominal bonds and linkers if you want to get the maximum break-even inflation (BEI) exposure?
- If you sell an amount of nominal bonds such that the duration is the same as the real duration of the linker, then if real rates increase the loss on the linker nets out with the gain in the short position in the nominal bond. If inflation expectations go up only the short position in the nominal bond is hit
- In our example buy 100 DGBi23 and sell 100*9.87/9.14=108 nominal bonds. Bloomberg uses bond risk to find the ratio . Type DK0009923054 <Corp> DK0009922916 <Corp> SS <go>. This will show hedge ratios from bond risk. Make sure to set Yield Beta = 1
- This hedge ratio gives maximal exposure to inflation expectations. In the first section we found the nominal duration which is an other exercise

Hedge ratios for optimal BEI exposure

$$dPV^{nom} = \frac{\partial PV}{\partial r} \cdot dr + \frac{\partial PV}{\partial \pi} \cdot d\pi$$

$$dPV^{ILB} = \frac{\partial PV}{\partial r} \cdot dr + 0 \cdot d\pi$$

Bloomberg calculation

Sell DGB 1 ½ 11/15/23 Corp | Buy DGBI 0.1 11/15/23 Corp | 98) Acti

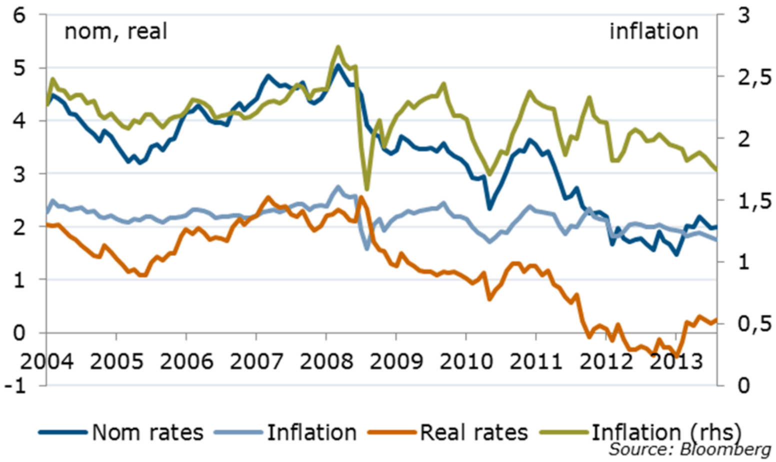
Pricing Source: My PCS | 95) Swap/Switch

Spread Summary	DGB 1 ½ 11/15/23		DGBI 0.1 11/15/23	
Hedge R	Sell	111	Buy	100
Price		97.814000		97.255000
Yield C		1.741156		0.381734
Spread Bnch		2.18		-133.77
Accrued Interest		0.078082		0.005205
Price + Accrued		97.892082		97.260205
Yield Beta				1.0
Dur/Risk/Cnv C		9.2997/8.9478/0.9615		9.9023/9.8998/1.0739
Moody's/S&P/Fitch		Aaa/AAA/AAA		Aaa/AAA/AAA
Settlement Date		12/04/13		12/04/13
Workout Dt/Price	W 11/15/23	100	W 11/15/23	100

What is the relation between i, r and π

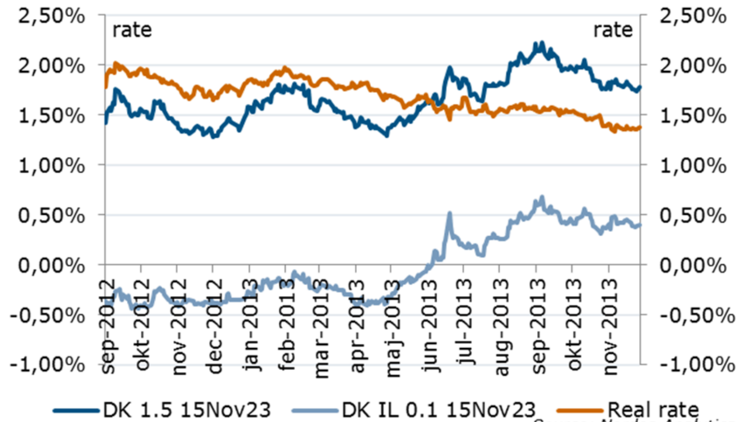
- Inflation expectations tend to be stable over time. This means that real rates and nominal rates are highly correlated. Further, until the ultra low rates from 2011 inflation was trending with the nominal (and real) levels
- It is important to stress that ILBs provide a real rate exposure as opposed to an inflation exposure. Real rates and inflation are not necessarily better correlated than nominal rates and inflation. Since May Danish nominal rates have increased, but real rates have increased *even more* and thus BEI has come down
- Now, it would be interesting to explore how the linker contributes to a nominal portfolio….
- Because of the asymmetry in the inflation exposure between nominal and real rate bonds, certain realistic macro scenarios give rise to an attractive risk-reward profile when adding the linker
- Basically we will be looking at the impact of different realized correlations between r and i on a portfolio that assumes a correlation of 0,75%

10y EUR nominal, real and inflation rates from IRS



Source: Bloomberg

Recent developments in DKK real and nominal rates

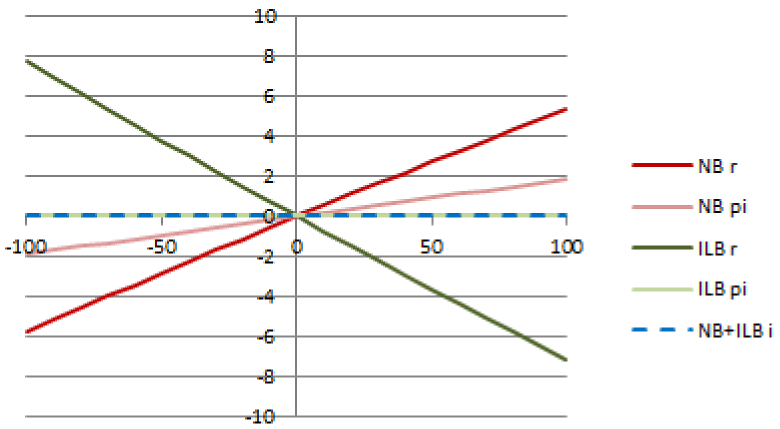


Source: Nordea Analytics

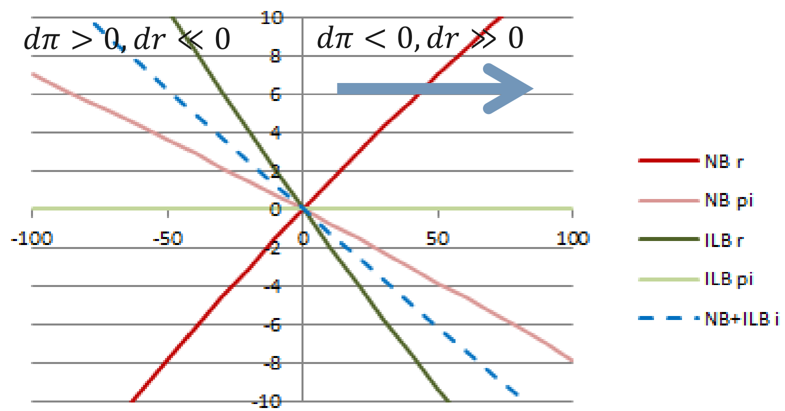
Linking the linker to a nominal sell-off

- In the following we show a 2nd order Taylor approximation of a long position in the linker and a short position in the nominal bond. The hedge ratio in all examples is set such that linker's nominal delta is hedged. I.e. the hedge complies with a 0.75 dr - di correlation. The position is showed as a dashed blue line in the charts
- The graphs show the PV / rates relation for the aforementioned position and we allow for the nominal rate changes to be decomposed into inflation and a real component
- In the first graph the relation between di and dr is *set* to 0.75 such that the empirical relation holds ex ante. This means that the total exposure of the position is zero (except for convexity) for all rate shifts. That is: NB+ILB is flat at zero
- The price effects from changes to real rates and inflation can be found in the appendix
- Since Q1 2013 inflation expectations have fallen and nominal rates increased. This is the $d\pi$<0, dr≫0 regime. The chart on the right shows an example of how that could look like. Real rates affect both the ILB and the NB but inflation only affects the (short) nominal bond

Hedged linker position in the $\frac{\partial r}{\partial i}$ = 0.75 regime



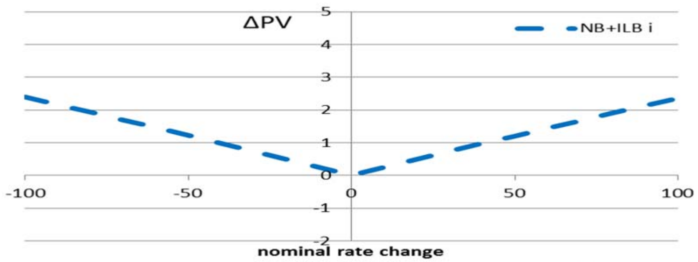
Hedged linker position in the current regime



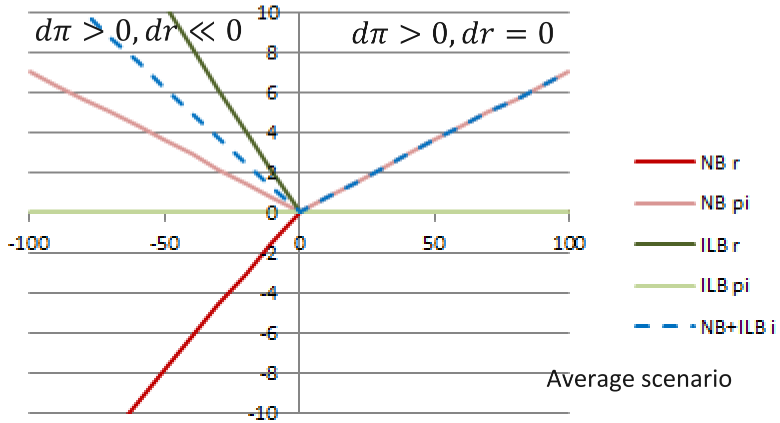
"NB r" is the real rate effect on nominal bond
"ILB r" is the real rate effect on linker bond etc.

Different real/nom correlation if rates rise or fall?

- What is a likely mix of real rates and inflation changes in a nominal rate increase?
- In the short run nominal rate increases are likely to be based on increased inflation expectations and more sticky real rates (unwinding the current regime). This could be **combined** with a continuation of the current regime should rates fall
- Lets assume, now, that inflation is back a 2% then…
- A plausible medium term scenario for increasing nominal rates could be that expected inflation stays at 2% and real rates increase from approx. 0 to 2%. That is by and large the $d\pi$=0, dr>0 scenario taking over from the $d\pi$>0, dr=0 scenario
- Now, if we believe in some combination of the above scenarios, where inflation expectations are to rise from the current level and that real rates eventually will go up with inflation expectations anchored around e.g. 2%, then the "average" of the blue dashed lines in the two graphs at the right indicates an attractive risk-reward or – if you like - "positive convexity" by adding linkers to your portfolio

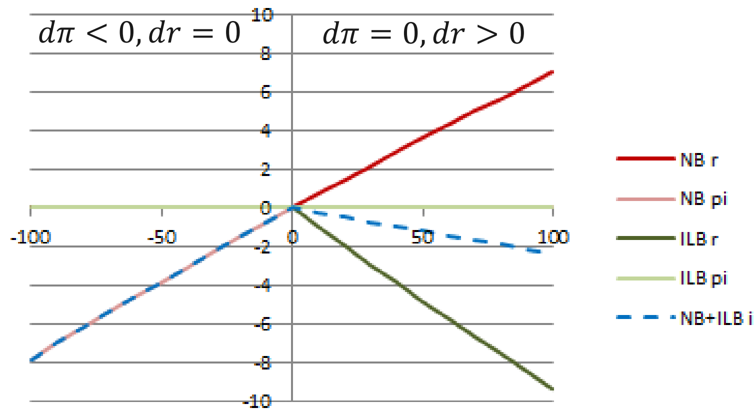


Inflation expectations drive nom. rates in up scenario



Average scenario

Inflation anchored at 2%, real rates go up



NB! There are a number of assumptions concerning probability and discounting in the above example

Appendix

The following table shows how a long position in ILB and a short position in the NB performs. (no roll, carry, liquidity etc. considerations). The weights are: +100 DGBi23 and -100· 7.5/ 9.87 = -76 DGB23. Cvx = 0 assumed

				ILB	ILB	NB	NB
				real	inflation	real	inflation
			BPV	-9,9	0	-9,3	-9,3
assumed dr/di	0,75		CVX	0	0	0	0
ILB nom delta	-7,425		Weight	100	100	-79,8	-79,8

	decomposed		example			ILB	ILB	short NB	short NB	position
nominal rates	real rates	inflation	dr	dπ	dr/di	dr	dπ	dr	dπ	"di"
di=0,10%			0,075%	0,025%	0,75	-0,74	-	0,56	0,19	-
di=0,10%	dr>0	dπ>0	0,05%	0,05%	0,50	-0,50	-	0,37	0,37	0,25
di=0,10%	dr=0	dπ>0	0,00%	0,10%	-	-	-	-	0,74	0,74
di=0,10%	dr>0	dπ=0	0,10%	0,00%	1,00	-0,99	-	0,74	-	-0,25
di=0,10%	dr>>0	dπ<0	0,20%	-0,10%	2,00	-1,98	-	1,49	-0,74	-1,24
di=0,10%	dr<0	dπ>>0	-0,10%	0,20%	-1,00	0,99	-	-0,74	1,49	1,73
di=0%	dr=0	dπ=0	0,00%	0,00%		-	-	-	-	-
di=0%	dr<0	dπ>0	-0,05%	0,05%		0,50	-	-0,37	0,37	0,50
di=0%	dr>0	dπ<0	0,05%	-0,05%		-0,50	-	0,37	-0,37	-0,50
di=-0,10%	dr<0	dπ<0	-0,05%	-0,05%	0,50	0,50	-	-0,37	-0,37	-0,25
di=-0,10%	dr=0	dπ<0	0,00%	-0,10%	-	-	-	-	-0,74	-0,74
di=-0,10%	dr<0	dπ=0	-0,10%	0,00%	1,00	0,99	-	-0,74	-	0,25
di=-0,10%	dr<<0	dπ>0	-0,20%	0,10%	2,00	1,98	-	-1,49	0,74	1,24
di=-0,10%	dr>0	dπ<<0	0,10%	-0,20%	-1,00	-0,99	-	0,74	-1,49	-1,73

Thank you!

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